**1** Here is a sector, $AOB$, of a circle with centre $O$ and angle $AOB = x°$

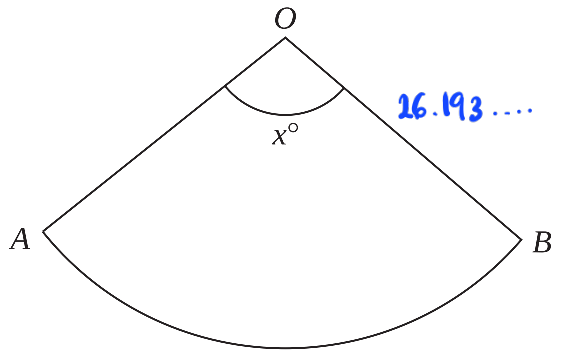



Diagram **NOT** accurately drawn

The sector can form the curved surface of a cone by joining $OA$ to $OB$.

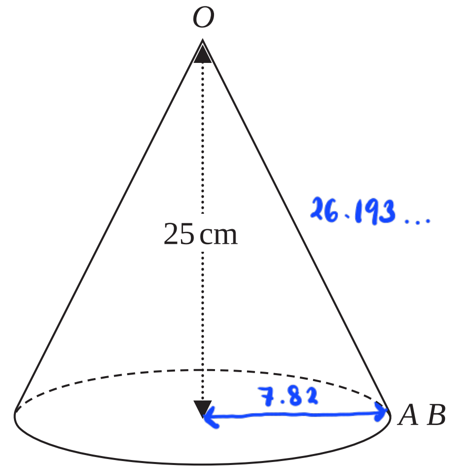



Diagram **NOT** accurately drawn

The height of the cone is 25 cm.
The volume of the cone is 1600 cm$^3$

Volume of Cone :

$\frac{1}{3} \times \pi \times r^2 \times h$

Work out the value of $x$.
Give your answer correct to the nearest whole number.

Finding radius of the cone :

$\frac{1}{3} \times \pi \times r^2 \times 25 = 1600$ (1)

$\pi r^2 = \frac{1600}{25} \times 3$

$r^2 = \frac{192}{\pi}$

$r = \sqrt{61.115}$

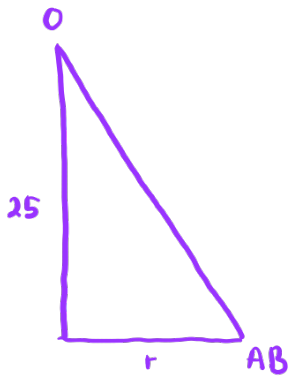
$= 7.8176 \ldots$ cm (1)

By using Pythagoras' theorem :

$OA^2 = 25^2 + 7.8176...^2$

$OA = \sqrt{686.1154...}$

$= 26.193....$ (1)




circumference of the circle :

$2\pi r = 2 \times \pi \times 7.8176...$

$= 49.1194...$ (1)

length of arc of the circle :

$2 \times \pi \times 26.193.... \times \frac{x}{360°} = 49.1194....$ (1)

$x = 107°$ (1)

$x =$ 107°

**(Total for Question 1 is 6 marks)**

**2** The diagram shows a triangular prism *ABCDEF* with a horizontal base *ABEF*.

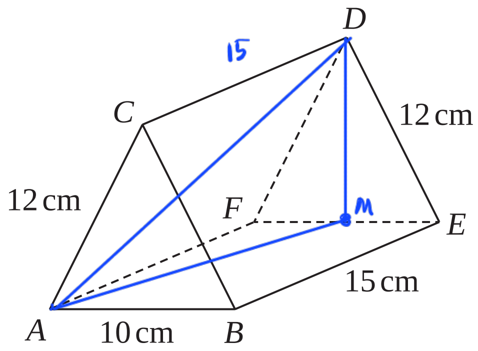



Diagram **NOT** accurately drawn

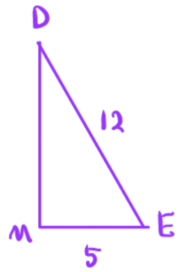
$AC = BC = FD = ED = 12$ cm $\quad AB = 10$ cm $\quad BE = 15$ cm

Calculate the size of the angle between *AD* and the base *ABEF*.
Give your answer correct to 3 significant figures.

By using Pythagoras' theorem:

$DM = \sqrt{12^2 - 5^2}$

$= \sqrt{119}$ (1)



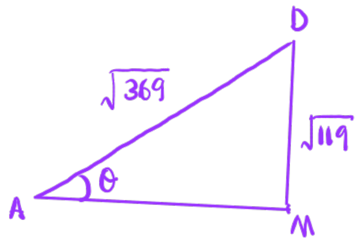

By using diagonal length length formula:

$AD = \sqrt{12^2 + 15^2}$

$= \sqrt{369}$ (1)

Angle between AD and base ABEF:

$\sin\theta = \frac{\sqrt{119}}{\sqrt{369}}$ (1)



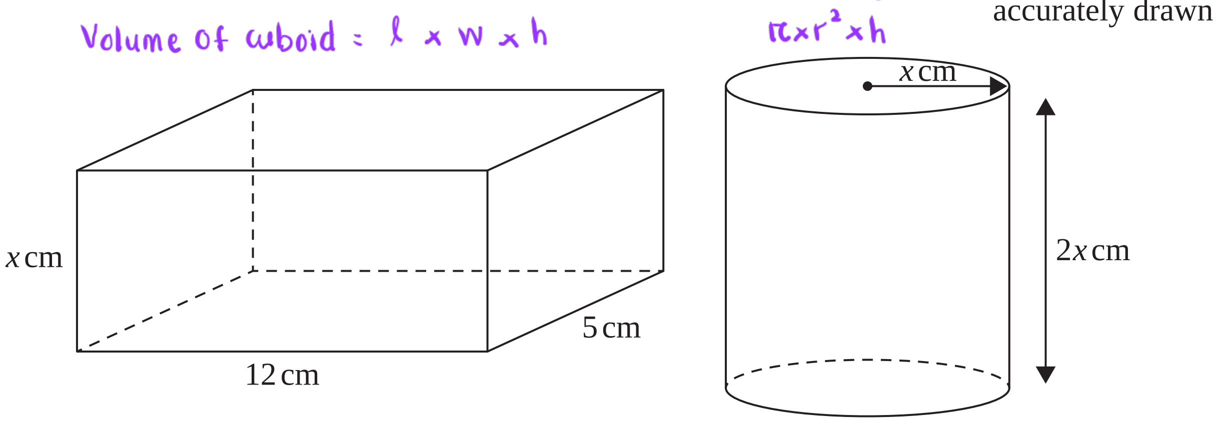

$\theta = \sin^{-1}\frac{\sqrt{119}}{\sqrt{369}}$

$= 34.6^\circ$ (1)

34.6 °

**(Total for Question 2 is 4 marks)**

**3** The diagram shows a cuboid and a cylinder.




The dimensions of the cuboid are $x$ cm by 12 cm by 5 cm.
The volume of the cuboid is 270 cm$^3$

The radius of the cylinder is $x$ cm.
The height of the cylinder is $2x$ cm.

(a) Work out the volume of the cylinder.
Give your answer correct to the nearest whole number.

Volume of cuboid = $12 \times 5 \times x = 270$

$60x = 270$

$x = \frac{270}{60}$

$= 4.5$ cm 1

Volume of cylinder = $\pi \times x^2 \times 2x$

$= \pi \times (4.5)^2 \times 2(4.5)$ 1

$= 573$ cm$^3$ 1

573 cm$^3$

**(3)**

(b) Change 1 m$^3$ to cm$^3$

$m \xrightarrow{\times 100} cm$, $cm \xrightarrow{\div 100} m$

$m^3 \xrightarrow{\times (100)^3} cm^3$, $cm^3 \xrightarrow{\div (100)^3} m^3$

$1\ m^3 \times \frac{(100)^3\ cm^3}{(1)^3\ m^3} = 1\ 000\ 000$ 1

1 000 000 cm$^3$

**(1)**

**(Total for Question 3 is 4 marks)**

**4** *ABC* is an isosceles triangle in a horizontal plane.
The point *T* is vertically above *B*.

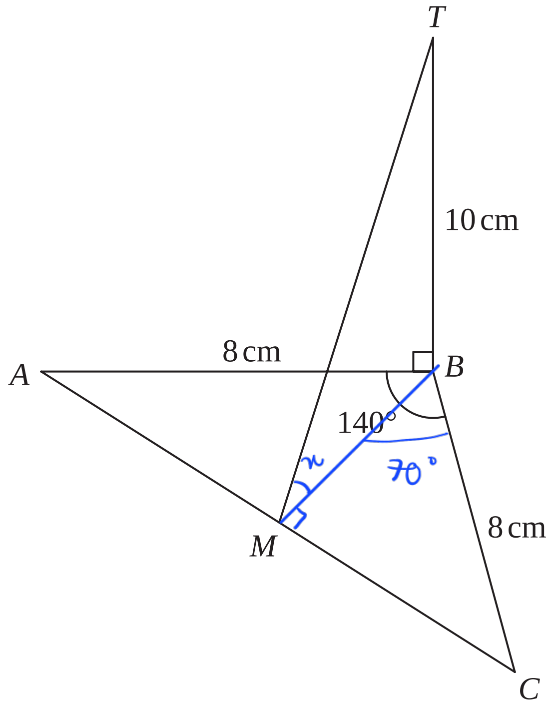
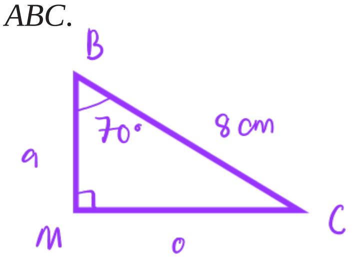



Diagram **NOT** accurately drawn

Angle $ABC = 140°$
$AB = BC = 8$ cm
$TB = 10$ cm
*M* is the midpoint of *AC*.

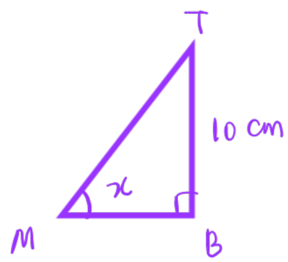
Calculate the size of the angle between *MT* and the horizontal plane *ABC*.
Give your answer correct to one decimal place.




$$\cos 70° = \frac{MB}{8} \quad (1)$$

$$MB = 8\cos 70°$$

$$= 2.73616\ldots \quad (1)$$




$$\tan x = \frac{10}{2.73616\ldots} \quad (1)$$

$$= 3.6547\ldots$$

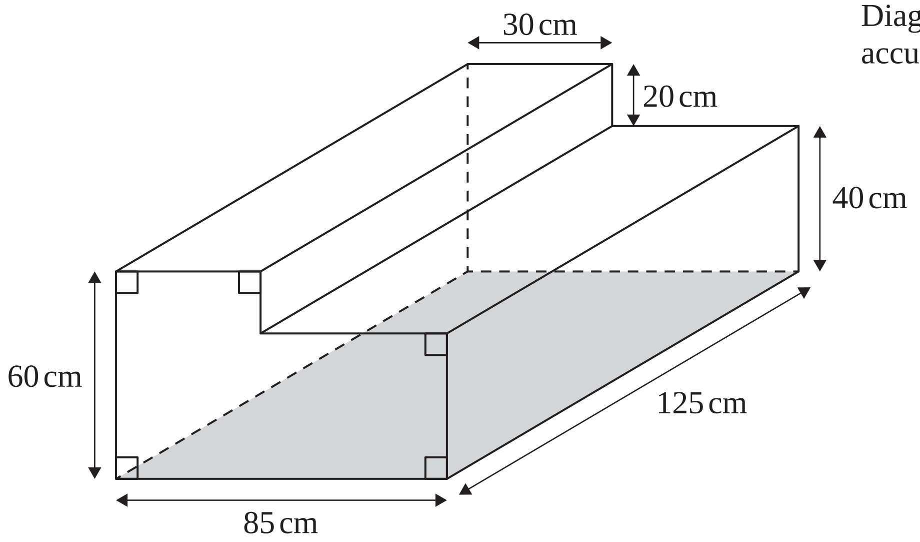
$$x = \tan^{-1}(3.6547\ldots)$$

$$= 74.7° \text{ (1dp)} \quad (1)$$

74.7 °

**(Total for Question 4 is 4 marks)**

**5** The diagram shows a container for water in the shape of a prism.




The rectangular base of the prism, shown shaded in the diagram, is horizontal.
The container is completely full of water.

Tuah is going to use a pump to empty the water from the container so that the volume of water in the container decreases at a constant rate.

The pump starts to empty water from the container at 10 30 and at 12 00 the water level in the container has dropped by 20 cm.

Find the time at which all the water has been pumped out of the container.

$85 \times 125 \times 40 = 425\,000 \text{ cm}^3$ (water left in container)

(1) $30 \times 20 \times 125 = 75\,000 \text{ cm}^3$ ( water that has been pumped out )

$$\frac{75\,000 \text{ cm}^3}{425\,000 \text{ cm}^3} = \frac{1.5 \text{ hour}}{x}$$

$$x = \frac{425\,000 \times 1.5}{75\,000} \quad (2)$$

$$= 8.5 \text{ hours}$$

$1200 + 8.5 \text{ hours} = 2030$ (1)

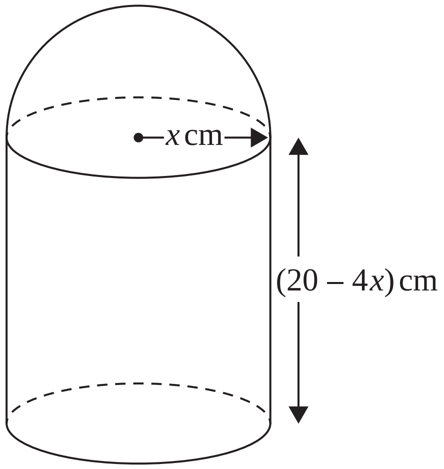
2030

**(Total for Question 5 is 4 marks)**

**6** A solid, **S**, is made from a hemisphere and a cylinder.

The centre of the circular face of the hemisphere and the centre of the top face of the cylinder are at the same point.

Diagram **NOT** accurately drawn




The radius of the cylinder and the radius of the hemisphere are both $x$ cm.
The height of the cylinder is $(20 - 4x)$ cm.

The volume of **S** is $V$ cm$^3$ where $V = \frac{1}{3}\pi y$

Find the maximum value of $y$.
Show clear algebraic working.

Volume of sphere $= \frac{4}{3}\pi r^3$ Volume of hemisphere $= \frac{2}{3}\pi r^3$

Volume of S $= \frac{2}{3}\pi r^3 + \pi r^2 h$

$= \frac{2}{3}\pi x^3 + \pi x^2 (20 - 4x)$

$= \frac{2}{3}\pi x^3 + 20\pi x^2 - 4\pi x^3$ (1)

$\frac{1}{3}\pi y = \frac{2}{3}\pi x^3 + 20\pi x^2 - 4\pi x^3$ (1)

$\frac{1}{3}\pi y = \left(\frac{2}{3}\pi - 4\pi\right) x^3 + 20\pi x^2$ (cancel out $\pi$)

$\frac{1}{3} y = \left(\frac{2}{3} - 4\right) x^3 + 20x^2$

$= -\frac{10}{3} x^3 + 20x^2$

$$\frac{y}{3} = -\frac{10}{3}x^3 + 20x^2$$

$$y = -10x^3 + 60x^2 \quad 1$$

$$\frac{dy}{dx} = -30x^2 + 120x$$

$$0 = -30x^2 + 120x \quad 1$$

$$30x^2 = 120x$$

$$x = \frac{120x}{30x}$$

$$x = 4$$

$$y = -10(4)^3 + 60(4)^2$$

$$= 320 \quad 1$$

320

**(Total for Question 6 is 5 marks)**

**7** The diagram shows a solid prism *ABCDEFGH*.

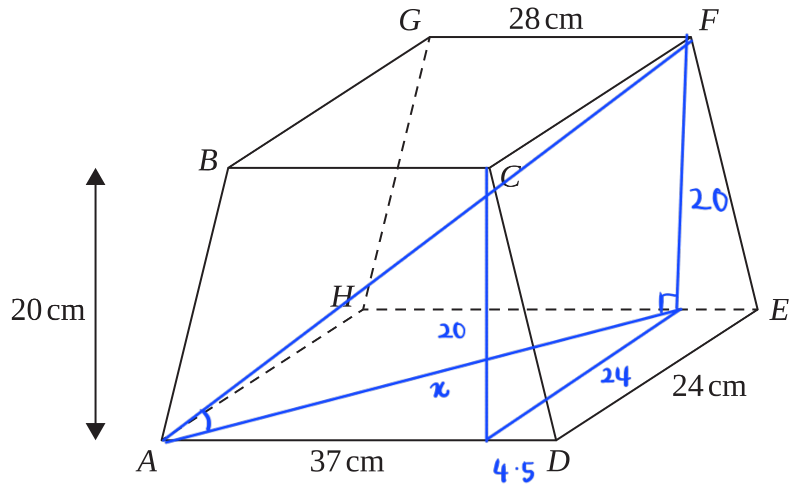



Diagram **NOT** accurately drawn

The trapezium *ABCD*, in which *AD* is parallel to *BC*, is a cross section of the prism.
The base *ADEH* of the prism is a horizontal plane.
*ADEH* and *BCFG* are rectangles.
The midpoint of *BC* is vertically above the midpoint of *AD* so that *BA* = *CD*.

*AD* = 37 cm *GF* = 28 cm *DE* = 24 cm

The perpendicular distance between edges *AD* and *BC* is 20 cm.

(a) Work out the total surface area of the prism.

$CD = \sqrt{4.5^2 + 20^2}$

$= 20.5\text{cm}$ (1)

(1)

Total surface area $= 2 \times \frac{1}{2} \times (37 + 28) \times 20 + 2 \times 24 \times 20.5 + 28 \times 24 +$

$24 \times 37$ (1)

$= 1300 + 984 + 672 + 888$

$= 3844 \text{ cm}^2$ (1)

3844 ........................................ cm²

**(4)**

(b) Calculate the size of the angle between *AF* and the plane *ADEH*.
Give your answer correct to one decimal place.

$x = \sqrt{(37 - 4.5)^2 + (24)^2}$

$= 40.4\ldots$ 1

$\tan \angle A = \dfrac{20}{40.4\ldots}$ 1

$\angle A = \tan^{-1}(0.495\ldots)$

$\angle A = 26.3°$ 1

26.3 °

(3)

**(Total for Question 7 is 7 marks)**

**8** The diagram shows a cube *ABCDEFGH* with sides of length 6 cm.

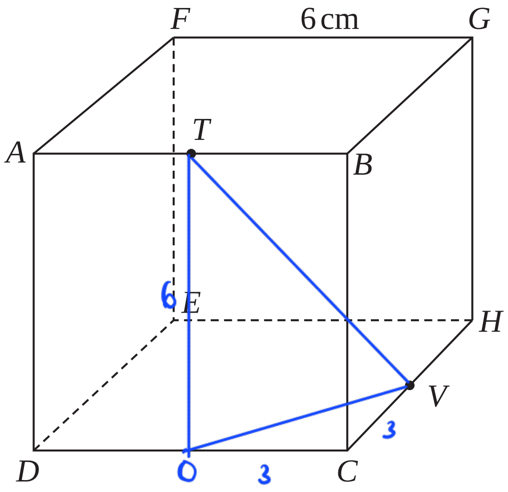
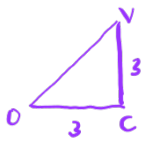



Diagram **NOT** accurately drawn

*T* is the midpoint of *AB* and *V* is the midpoint of *CH*

Work out the distance from *T* to *V* in a straight line through the cube.
Give your answer in the form $\sqrt{a}$ cm where *a* is an integer.

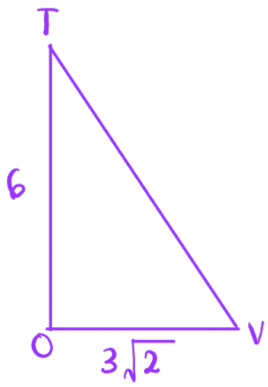
Finding length OV :




$OV = \sqrt{3^2 + 3^2}$ (1)

$= 3\sqrt{2}$ cm (1)

Finding length TV :




$TV = \sqrt{6^2 + (3\sqrt{2})^2}$ (1)

$= \sqrt{54}$ (1)

where $a = 54$

$\sqrt{54}$ .......................................... cm

**(Total for Question 8 is 4 marks)**

**9** The diagram shows a frustum of a cone, and a sphere.

The frustum, shown shaded in the diagram, is made by removing the small cone from the large cone.
The small cone and the large cone are similar.

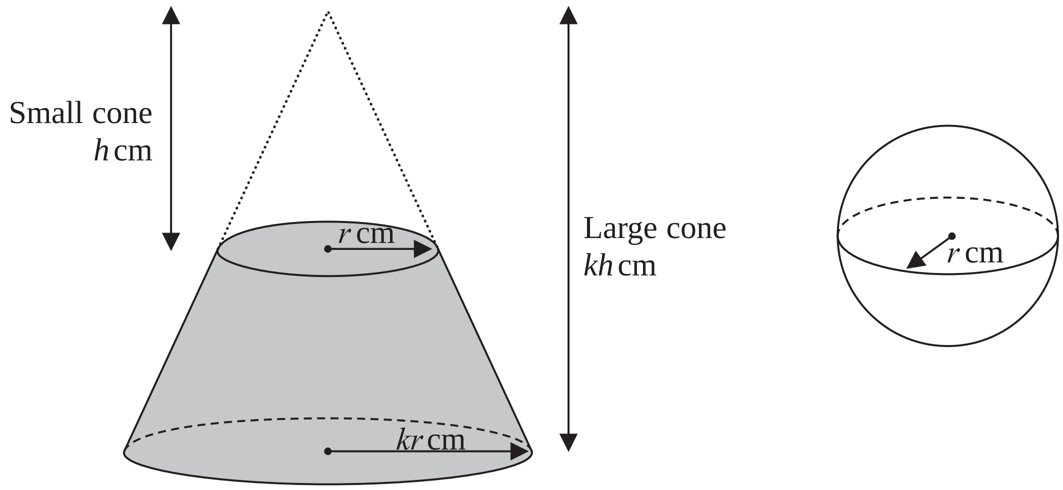



Diagram **NOT** accurately drawn

The height of the small cone is $h$ cm and the radius of the base of the small cone is $r$ cm.
The height of the large cone is $kh$ cm and the radius of the base of the large cone is $kr$ cm.
The radius of the sphere is $r$ cm.

The sphere is divided into two hemispheres, each of radius $r$ cm.

Solid **A** is formed by joining one of the hemispheres to the frustum.
The plane face of the hemisphere coincides with the upper plane face of the frustum, as shown in the diagram below.

Solid **B** is formed by joining the other hemisphere to the small cone that was removed from the large cone.
The plane face of the hemisphere coincides with the plane face of the base of the small cone, as shown in the diagram below.

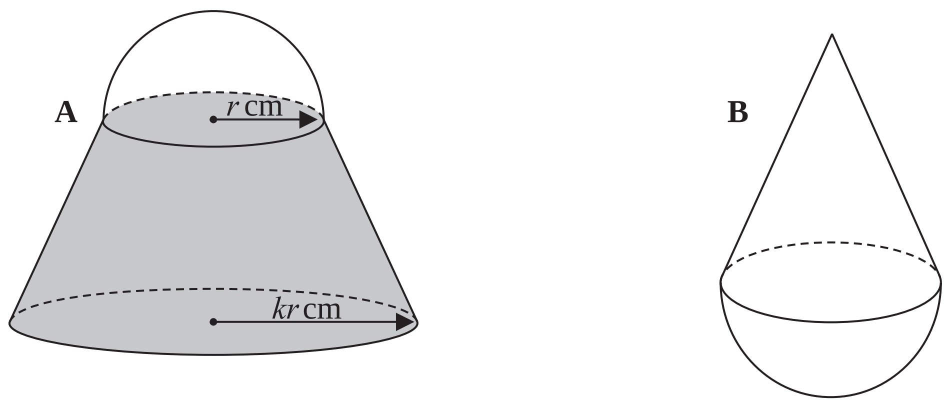



Diagram **NOT** accurately drawn

The volume of solid **A** is 6 times the volume of solid **B**.

Given that $k > \sqrt[3]{7}$

find an expression for $h$ in terms of $k$ and $r$

Volume of each hemisphere :

$\frac{1}{2} \times$ volume of sphere $= \frac{1}{2} \times \frac{4}{3} \times \pi \times r^3$

$= \frac{2}{3}\pi r^3$ 1

Volume of small cone :

$= \frac{1}{3}\pi r^2 h$

Volume of frustrum :

Volume of large cone - volume of small cone :

$\frac{1}{3} \times \pi \times (kr)^2 \times kh - \frac{1}{3} \times \pi \times r^2 \times h$

$= \frac{1}{3}\pi r^2 h(k^3 - 1)$ 1

Volume of Solid A :

$= \frac{1}{3}\pi r^2 h(k^3-1) + \frac{2}{3}\pi r^3$

Volume of Solid B :

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$ 1

$\therefore \frac{1}{3}\pi r^2 h(k^3-1) + \frac{2}{3}\pi r^3 = 6\left(\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right)$ 1

$\frac{1}{3}\pi r^2 h(k^3-1) + \frac{2}{3}\pi r^3 = 2\pi r^2 h + 4\pi r^3$

$\div \pi r^2$

$\frac{1}{3} h(k^3-1) + \frac{2}{3} r = 2h + 4r$

$h(k^3-1) + 2r = 6h + 12r$

$h(k^3-1) - 6h = 10r$ 1

$hk^3 - 7h = 10r$

$h = \frac{10r}{k^3-7}$ 1

$h = \frac{10r}{k^3-7}$

**(Total for Question 9 is 6 marks)**

**10** A solid is made from a cone and a hemisphere.

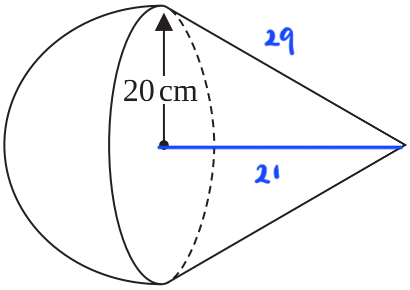



Diagram **NOT** accurately drawn

The circular plane face of the hemisphere coincides with the circular base of the cone.
The radius of the hemisphere and the radius of the circular base of the cone are both 20 cm.

The curved surface area of the cone is $580\pi\,\text{cm}^2$

The volume of the solid is $k\pi\,\text{cm}^3$

Work out the exact value of $k$

$580\pi = \pi \times 20 \times l$ 1

$l = \dfrac{580\pi}{20\pi}$

$= 29$ 1

$= \sqrt{29^2 - 20^2} = 21$ 1

$\left(\frac{1}{2} \times \frac{4}{3} \times \pi \times 20^3\right) + \left(\frac{1}{3} \times \pi \times 20^2 \times 21\right)$

$= \dfrac{16\,000}{3}\pi + \dfrac{8400}{3}\pi$ 1

$= \dfrac{24\,400}{3}\pi$

1

$k = \dfrac{24400}{3}$

**(Total for Question 10 is 5 marks)**

**11** The diagram shows a solid made from a cylinder and a hemisphere.
The cylinder and the hemisphere are both made from the same metal.

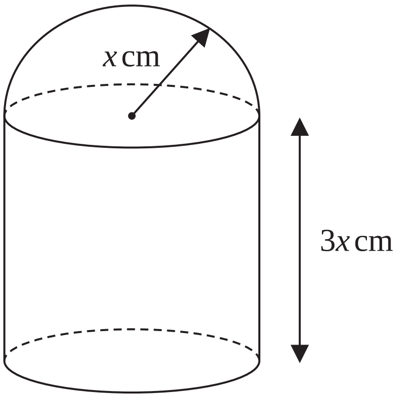



Diagram **NOT** accurately drawn

The plane face of the hemisphere coincides with the upper plane face of the cylinder.

The radius of the cylinder and the radius of the hemisphere are both $x$ cm.
The height of the cylinder is $3x$ cm.

The total surface area of the solid is $81\pi$ cm$^2$
The mass of the solid is 840 grams.

The following table gives the density of each of four metals.

| **Metal** | **Density (g/cm$^3$)** |
|---|---|
| Aluminium | 2.7 |
| Nickel | 8.9 |
| Gold | 19.3 |
| Silver | 10.5 |

The metal used to make the solid is one of the metals in the table.

Determine the metal used to make the solid.
Show your working clearly.

$\pi x^2 + 2\pi x \times 3x + \frac{1}{2} \times 4\pi x^2 = 81\pi$ 1

$\pi x^2 + 6\pi x^2 + 2\pi x^2 = 81\pi$

$9\cancel{\pi}x^2 = 81\cancel{\pi}$

$9x^2 = 81$

$x^2 = 9$ 1

$x = 3$

Volume : $\pi \times 3^2 \times 3(3) + \frac{1}{\cancel{2}} \times \frac{\cancel{4}^{2}}{3} \times \pi (3)^3$ (1)

$= 81\pi + 18\pi = 99\pi$ (1)

$\frac{840}{99\pi}$ (1) $= 2.7....$ (aluminium)

(1) aluminium

**(Total for Question 11 is 6 marks)**

**12** The diagram shows a solid triangular prism.

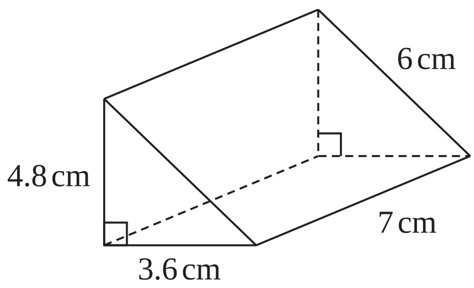



Diagram **NOT** accurately drawn

Work out the **total** surface area of the triangular prism.
Give your answer correct to 3 significant figures.

$\left(2 \times \frac{1}{2} \times 4.8 \times 3.6\right) + (7 \times 6) + (7 \times 3.6) + (4.8 \times 7)$ 1

$= 17.28 + 42 + 25.2 + 33.6$ 1

$= 118.08$

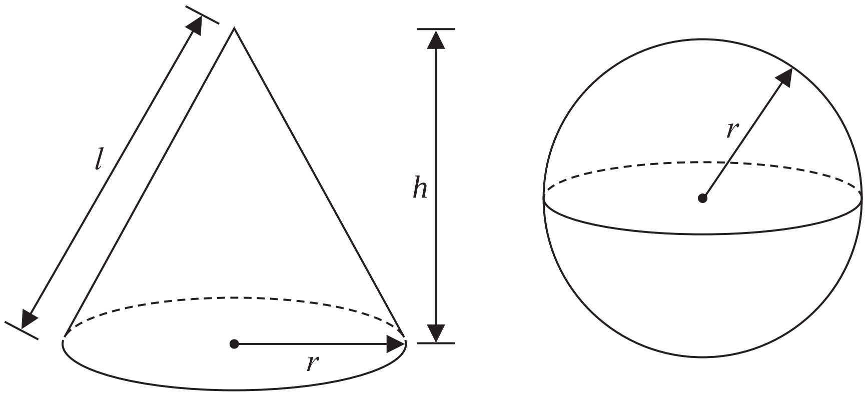
$\approx 118$ 1

118 cm$^2$

**(Total for Question 12 is 3 marks)**

**13** The diagram shows a solid cone and a solid sphere.

Diagram **NOT** accurately drawn




The cone has base radius $r$, slant height $l$ and perpendicular height $h$
The sphere has radius $r$

The base radius of the cone is equal to the radius of the sphere.

Given that

$$k \times \text{volume of the cone} = \text{volume of the sphere}$$

show that the **total** surface area of the cone can be written in the form

$$\pi r^2 \left( \frac{k + \sqrt{k^2 + a}}{k} \right)$$

where $a$ is a constant to be found.

$k \times \frac{1}{\cancel{3}} \times \cancel{\pi} \times \cancel{r^2} \times h = \frac{4}{\cancel{3}} \cancel{\pi} \times \cancel{r^3}$ (1)

$kh = 4r$

$h = \frac{4r}{k}$ (1)

$l^2 = h^2 + r^2$

$= \left(\frac{4r}{k}\right)^2 + r^2$ (1)

$l = \sqrt{r^2 + \left(\frac{4r}{k}\right)^2}$

$$l = \sqrt{r^2\left(1 + \frac{16}{k^2}\right)}$$

$$l = r\sqrt{\frac{k^2+16}{k^2}}$$

$$l = r\frac{\sqrt{k^2+16}}{k} \quad (1)$$

Total surface area $= \pi r^2 + \pi r l$

$$= \pi r^2\left(1 + \frac{\sqrt{k^2+16}}{k}\right) \quad (1)$$

$$= \pi r^2\left(\frac{k + \sqrt{k^2+16}}{k}\right) \quad (1)$$

**(Total for Question 13 is 6 marks)**

**14** Given that the surface area of a sphere is $49\pi\,\text{cm}^2$

find the volume of the sphere.
Give your answer correct to the nearest integer.

$4\pi r^2 = 49\pi$

$r^2 = \frac{49\pi}{4\pi}$ (1)

$r = \sqrt{\frac{49\cancel{\pi}}{4\cancel{\pi}}}$

$= \frac{7}{2} = 3.5$

$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 3.5^3$ (1)

$= 180$ (1)

180 $\text{cm}^3$

**(Total for Question 14 is 3 marks)**